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a $f(g(x))$

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Self Tutor

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a

$$\begin{aligned} & f(g(x)) \\ &= f(x^2+4) \\ &= 3(x^2+4)+2 \\ &= 3x^2+12+2 \\ &= 3x^2+14 \end{aligned}$$

b

$$\begin{aligned} & g(f(x)) \\ &= g(3x+2) \\ &= (3x+2)^2+4 \\ &= 9x^2+12x+4+4 \\ &= 9x^2+12x+8 \end{aligned}$$

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To find $f(g(x))$ we look at the f function. Whenever we see x we replace it by $g(x)$ within brackets.

From the previous **Example**, we can see that in general, $f(g(x)) \neq g(f(x))$.

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Self Tutor

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a

$$\begin{aligned} &f(f(x)) \\ &= f(2x-1) \\ &= 2(2x-1)-1 \\ &= 4x-2-1 \\ &= 4x-3 \end{aligned}$$

b

$$\begin{aligned} &f(f(5)) \\ &= 4(5)-3 \qquad \{\text{using } \mathbf{a}\} \\ &= 17 \end{aligned}$$

EXERCISE 15D

1 If $f(x) = 3x - 4$ and $g(x) = 2 - x$, find in simplest form:

- **a** $f(g(x))$
- **b** $g(f(x))$
- **c** $f(f(x))$
- **d** $g(g(x))$

2 If $f(x) = \sqrt{x}$ and $g(x) = 4x - 3$, find in simplest form:

- **a** $f(g(x))$
- **b** $g(f(x))$
- **c** $f(g(7))$
- **d** $g(f(4))$

3 Find two functions f and g such that:

a $f(g(x)) = \sqrt{x-3}$

b $f(g(x)) = (x+5)^3$

c $f(g(x)) = \dfrac{5}{x+7}$

d $g(f(x)) = \dfrac{1}{\sqrt{3-4x}}$

e $g(f(x)) = 3^{x^2}$

f $g(f(x)) = \left(\dfrac{x+1}{x-1}\right)^2$

4 Suppose $f(x) = 3x + 1$ and $g(x) = x^2 + 2x$.

a Find $f(g(x))$.

b Find x such that $f(g(x)) = 10$.

5 Suppose $f(x) = 4x + 1$ and $g(x) = \dfrac{1}{x+2}$.

a Find $f(g(x))$.

b Find $f(g(-1))$.

c Find x such that $f(g(x)) = 3$.

6 Suppose $f(x) = 2x + 5$ and $g(x) = \dfrac{x-5}{2}$.

a Find:

- **i** $f(4)$
- **ii** $g(13)$
- **iii** $g(17)$
- **iv** $f(6)$.

b Find $f(g(x))$ and $g(f(x))$.

c Find:

- **i** $f(g(3))$
- **ii** $g(f(-7))$.

EXERCISE 15D

1 **a** $2-3x$ **b** $6-3x$ **c** $9x-16$ **d** x

2 **a** $\sqrt{4x-3}$ **b** $4\sqrt{x}-3$ **c** 5 **d** 5

3 **a** $f(x)=\sqrt{x},\ g(x)=x-3$ **b** $f(x)=x^3,\ g(x)=x+5$

c $f(x)=\dfrac{5}{x},\ g(x)=x+7$ **d** $f(x)=3-4x,\ g(x)=\dfrac{1}{\sqrt{x}}$

e $f(x)=x^2,\quad g(x)=3^x$ **f** $f(x)=\dfrac{x+1}{x-1},\quad g(x)=x^2$

(**Note:** There may be other answers.)

4 **a** $f(g(x))=3x^2+6x+1$ **b** $x=-3$ or 1

5 **a** $f(g(x))=\dfrac{4}{x+2}+1\quad\left(\text{or }\dfrac{x+6}{x+2}\right)$ **b** 5 **c** $x=0$

6 **a** **i** 13 **ii** 4 **iii** 6 **iv** 17

b $f(g(x))=x$ and $g(f(x))=x$ **c** **i** 3 **ii** -7